YÜZEYLER TEORİSİ QUİZ SINAV SORULARI (12.11.2018)

Adı Soyadı:

Numarası:

1	2	Toplam

1.) $M_1 = \{ (x_1, x_2, x_3) \mid x_1^2 + x_2^2 = 4 \} \subset E^3$,

$M_2 = \{ (x_1, x_2, x_3) \mid x_1^2 - x_2^2 - 2x_3 = 0 \} \subset E^3$

yüzeylerinin arakesiti olan eğrinin $Q = (1, \sqrt{3}, ?)$ noktasındaki eğriliklerini bulunuz.

2.) E^3 de bir yüzey M olsun. M nin parametrik ifadesi

$$\begin{aligned} \Phi : I \times J \subset E^2 &\to E^3 \\ (u, v) &\to \Phi(u,v) = (\varphi_1(u,v), \varphi_2(u,v), \varphi_3(u,v)) \end{aligned}$$

olsun. $\{\Phi_u, \Phi_v\}$ sistemi $\chi(M)$ için bir ortogonal baz olsun. $V_1 = \frac{\Phi_u}{\|\Phi_u\|}$, $V_2 = \frac{\Phi_v}{\|\Phi_v\|}$ olmak üzere M yüzeyinin birim normal vektör alanı

$N = V_1 \wedge V_2 = \frac{1}{\|\Phi_u\| \|\Phi_v\|} \Phi_u \wedge \Phi_v$ olsun. Bu durumda, $\frac{dN}{du}$ nin en sade şeklini hesaplayınız.

NOT: Sorular eşit puanlı olup, süre 45 dakikadır.

Prof. Dr. Ayhan TUTAR

CEVAPLAR

C-1 $M_1=\{(x_1,x_2,x_3) \mid x_1^2+x_2^2=4\}\subset E^3$,
$M_2=\{(x_1,x_2,x_3) \mid x_1^2-x_2^2-2x_3=0\}\subset E^3$
yüzeylerinin arakesiti olan eğrinin $Q=(1,\sqrt{3},?)$ noktasındaki eğriliklerini bulunuz.

ÇÖZÜM: $x_1^2+x_2^2=4 \Rightarrow x_1=2\cos t,\ x_2=2\sin t$ yazabiliriz.

$x_1^2-x_2^2-2x_3=0 \Rightarrow 4\cos^2 t-4\sin^2 t=2x_3$

$$\Rightarrow x_3=2\cos^2 t-2\sin^2 t=2(\underbrace{\cos^2 t-\sin^2 t}_{\cos 2t})$$

$$\Rightarrow x_3=2\cos 2t.$$

Buna göre; M_1 ve M_2 yüzeylerinin arakesit eğrisinin bir parametrizasyonu

$$\alpha: I \longrightarrow M_1\cap M_2\subset E^3$$
$$t \longrightarrow \alpha(t)=(2\cos t,\ 2\sin t,\ 2\cos 2t)$$

dir. Buradan,

$$\left.\begin{array}{l} x_1(P)=1 \Rightarrow 1=2\cos t \Rightarrow \cos t=1/2 \\ x_2(P)=\sqrt{3} \Rightarrow \sqrt{3}=2\sin t \Rightarrow \sin t=\sqrt{3}/2 \end{array}\right\} \Rightarrow t=\frac{\pi}{3}.$$

$x_3(P)=2\cos 2.\frac{\pi}{3}=2\cos\frac{2\pi}{3}=2.\left(-\frac{1}{2}\right)=-1$ dir. O halde, $Q=(1,\sqrt{3},-1)$ olarak elde edilir.

$\alpha'(t)=(-2\sin t,\ 2\cos t,\ -4\sin 2t) \Rightarrow \alpha'(\pi/3)=(-\sqrt{3},1,-2\sqrt{3})$,

$\alpha''(t)=(-2\cos t,\ -2\sin t,\ -8\cos 2t) \Rightarrow \alpha''(\pi/3)=(-1,-\sqrt{3},4)$,

$\alpha'''(t)=(2\sin t,\ -2\cos t,\ 16\sin 2t) \Rightarrow \alpha''(\pi/3)=(\sqrt{3},-1,8\sqrt{3})$.

$$k_1(\pi/3)=\frac{\|\alpha'(\pi/3)\times\alpha''(\pi/3)\|}{\|\alpha'(\pi/3)\|^3} \quad \text{ve} \quad k_2(\pi/3)=\frac{\det(\alpha'(\pi/3),\alpha''(\pi/3),\alpha'''(\pi/3))}{\|\alpha'(\pi/3)\times\alpha''(\pi/3)\|^2}$$

formüllerinden $k_1(\pi/3)$ ve $k_2(\pi/3)$ değerlerini hesaplayalım.

$$\alpha'(\pi/3)\times\alpha''(\pi/3)=\begin{vmatrix} e_1 & e_2 & e_3 \\ -\sqrt{3} & 1 & -2\sqrt{3} \\ -1 & -\sqrt{3} & 4 \end{vmatrix}=-2e_1+6\sqrt{3}e_2+4e_3$$

$$\Rightarrow \|\alpha'(\pi/3)\times\alpha''(\pi/3)\|=\sqrt{(-2)^2+(6\sqrt{3})^2+4^2}=\sqrt{128}=\sqrt{2.64}=8\sqrt{2}.$$

$$\| \alpha'(\pi/3) \| = \sqrt{16} = 4$$

$$\det(\alpha'(\pi/3), \alpha''(\pi/3), \alpha'''(\pi/3)) = \langle \alpha'(\pi/3) \wedge \alpha''(\pi/3), \alpha'''(\pi/3) \rangle$$

$$= \langle (-2, 6\sqrt{3}, 4), (\sqrt{3}, -1, 8\sqrt{3}) \rangle$$

$$= -2\sqrt{3} - 6\sqrt{3} + 32\sqrt{3}$$

$$= 24\sqrt{3}.$$

$$k_1(\pi/3) = \frac{8\sqrt{2}}{16 \cdot 4} = \frac{\sqrt{2}}{8},$$

$$k_2(\pi/3) = \frac{\overset{3}{\cancel{24}}\sqrt{3}}{\underset{8}{\cancel{64}} \cdot 2} = \frac{3\sqrt{3}}{16}$$ bulunur.

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C-2)

$$N = V_1 \wedge V_2 = \frac{1}{\|\Phi_u\| \, \|\Phi_v\|} \Phi_u \wedge \Phi_v$$ dir. Buradan,

$$N = (\Phi_u \wedge \Phi_v)\left[\langle \Phi_u, \Phi_u \rangle^{-1/2} \cdot \langle \Phi_v, \Phi_v \rangle^{-1/2}\right]$$ yazılabilir.

$$\frac{dN}{du} = \frac{\Phi_{uu} \wedge \Phi_v + \Phi_u \wedge \Phi_{vu}}{\|\Phi_u\| \cdot \|\Phi_v\|} - \Phi_u \wedge \Phi_v \, \frac{1}{2} \, \frac{\langle \Phi_{uu}, \Phi_u \rangle + \langle \Phi_u, \Phi_{uu} \rangle}{\langle \Phi_u, \Phi_u \rangle^{3/2} \cdot \langle \Phi_u, \Phi_u \rangle^{1/2}}$$

$$- \Phi_u \wedge \Phi_v \cdot \frac{1}{2} \, \frac{\langle \Phi_{vu}, \Phi_v \rangle + \langle \Phi_v, \Phi_{vu} \rangle}{\langle \Phi_v, \Phi_v \rangle^{3/2} \cdot \langle \Phi_u, \Phi_u \rangle^{1/2}}$$

$$\Rightarrow \frac{dN}{du} = \frac{\Phi_{uu} \wedge \Phi_v + \Phi_u \wedge \Phi_{vu}}{\|\Phi_u\| \cdot \|\Phi_v\|} - \Phi_u \wedge \Phi_v \, \frac{\langle \Phi_{uu}, \Phi_u \rangle}{\left(\|\Phi_u\|^2\right)^{3/2} \cdot \|\Phi_v\|}$$

$$- \Phi_u \wedge \Phi_v \, \frac{\langle \Phi_{vu}, \Phi_v \rangle}{\underbrace{\left(\|\Phi_v\|^2\right)^{3/2}}_{\|\Phi_v\|^3} \cdot \|\Phi_u\|}$$